Computing the true spread*

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Abstract

How much of the observed labour productivity spread is real? This paper develops a novel framework to calculate productivity differences between plants which are due to differences in TFP and/or idiosyncratic demand shocks and not due to measurement error in variable factors or substitution between labour and other factors. The framework simultaneously accounts for imperfect competition, variations in output prices across plants and endogeniety of factor inputs. For UK manufacturing as a whole I find that on average 59 percent of the labour productivity spread is explained by TFP and demand shocks. Measurement error accounts on average for 9 percent of labour productivity spreads. This masks considerable heterogeneity across 3 digit sectors with measurement error accounting for as much as 28 percent in some sectors.

1 Introduction

A number of studies (these include Haskel and Martin [5], Oulton [13], Bailey et al [1], Bartelsman and Dhrymes [2]) have revealed that plant level data

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displays huge variations in measured productivity even between firms that populate the same 4 digit industry cluster. For example in terms output per employee the ratio between firms at the 90iest and 10th is on average more than 5 [5]. This phenomenon has been referred to as the productivity spread There are two basic hypothesis about the productivity spread: First, in a free market economy productivity spreads are something normal and simply a byproduct of the competitive selection process between good and bad firms. Alternatively, productivity spreads might be an issue of concern reflecting a for some reason hampered selection process where low productivity firms do not exit but bind resources in an unproductive way.

A slightly different hypothesis is that productivity spreads do not really exist but are rather the outcome of measurement problems. The classical example concerns labour inputs: Different plants might simply substitute between low and high skilled labour. High skilled workers work better and therefore less of them are needed but they also cost more. The plant employing the high skilled workers will have a higher measured productivity but there is no way in which shifting the low skilled workers to the high skilled plant will improve overall productivity because that plant does not have any superior technology either to get more out of them.

A similar argument can be made regarding material inputs. Some plants could decide to use fewer but higher quality intermediates achieving the same results as plants which employ more of the cheaper variety. As we do not have firm level intermediates prices we are ignorant of this, however.

Before deciding if there is too much or too little of the right or wrong spread it is therefore worthwhile to improve our measure of the spread.

To make progress note that all the measurement problems described above while affecting measures of productivity have no effect on measured profits. Profits are however correlated with genuine variations in TFP. The following section develops an estimation framework that exploits this observation to come up with a measure of the true spread.

2 A model to estimate the true spread

Before deriving the model note that genuine variations in TFP are not compatible with perfect competition. If one plant in the market has a technology which – using the same inputs – can produce more output, then this plant can always cut its price a little to drive its competitors with the inferior technology out of the market. It is funny however that most approaches to measure TFP at the plant level start with the assumption that the market structure can be characterized by perfect competition. Why bother to measure plant level TFP if you believe in perfect competition? The answer is that nobody believes in perfect competition but in order to measure TFP you need plant level prices. But, since normally only sector level prices are available assuming perfect competition ensures that plant level prices are equal to sector level prices.

To avoid such inconsistencies I start here with what we really observe building on work by Klette and Grilliches [7]. What we really observe at the plant level is revenue deflated with an industry price index:

$$r_{it} - p_t = q_{it} + p_{it} - p_t \tag{1}$$

where everything is stated in log deviations from the industry median corresponding to the focus of the current paper on productivity variations within an industry. The median deviation notation also allows to ignore all terms which are constant accross all firms in an industry at given point in time such as the industry price index for example so that $p_t = 0$. To proceed we must introduce assumptions regarding technology and market structure.

For the production function assume that it is of a general form but homogenous to degree γ, i.e.

$$Q_{it} = A_{it} \left(f(\mathbf{X}_{it}) \right)^{\gamma} \tag{2}$$

where $f(\cdot)$ is a general differentiable linear homogenous function, A_{it} is a Hicks neutral shift parameter[1] and $\mathbf{X}_{it}$ is a vector of J inputs. Avoiding any further assumptions on the form[2] of the production function we can invoke the mean value theorem to write a plants output relative to the median plant as

$$q_{it} = a_{it} + \sum_{z=1}^{Z} \alpha_z x_{zit} \tag{3}$$

where

$$\alpha_z = \gamma f_z(\bar{\mathbf{X}}_{it}) \frac{\bar{X}_{zit}}{f(\bar{\mathbf{X}}_{it})} \tag{4}$$

$f_z(\cdot)$ denotes the partial derivative of $f(\cdot)$ with respect to factor z and $\bar{\mathbf{X}}_{it}$ is some point in the convex hull spanned by $\mathbf{X}_{it}$ and $\mathbf{X}_{it-1}$.

For the market structure I follow Klette and Grilliches and assume a Dixit-Stiglitz monopolistic competition setting; i.e. plant level product demand becomes

$$Q_{it} = \frac{R_t}{P_t} \Lambda_{it}^{\eta-1} \left(\frac{P_{it}}{P_t} \right)^{-\eta} \tag{5}$$

where R_t is the sectoral revenue, P_t the sectoral price index and P_{it} the price of the individual firm. This is quite restrictive but better than ignoring the problem of price variation completely and not any more restrictive than the common practice to assume that everything in this world is produced by a Cobb Douglas production function.

A demand function such as Equation 5 implies a markup pricing rule

$$P_{it} \gamma \frac{Q_{it}}{f(\mathbf{X}_{it})} f_z(\mathbf{X}_{it}) = \mu W_{zit} \tag{6}$$

i.e. prices must be such that the marginal value product is μ times the marginal cost of each factor. Our demand function implies that $\mu = \frac{1}{1-\frac{1}{\eta}}$.

As pointed out by Klette[8], Equation 6 can only be expected to hold for production factors which are easily adjustable. I distinguish in the following between 3 types of inputs: labour, intermediates and capital. I assume

[1] Also known as TFP

[2] Except for differentiability that is

labor and material can be adjusted immediately to it optimal value while capital is fixed in the short term. As a consequence 6 holds in for labour L and intermediates M conditional on the level of the capital stock K. For intermediates and labour we can therefore write

$$\alpha_j = \mu \frac{W_j X_{jit}}{P_{it} Q_{it}} = \mu s_{jit} \tag{7}$$

where S_{jit} is the revenue share of factor j. Further because we assumed linear homogeniety of $f(\cdot)$ we get that

$$\alpha_K = \gamma - \alpha_L - \alpha_M \tag{8}$$

We get therefore in 3 that

$$q_{it} = a_{it} + \mu v i_{it} + \mu \varsigma_{it} + \gamma k_{it} \tag{9}$$

where

$$v i_{it} = \sum_{z \neq K} \bar{s}_j (x_{zit} - k_{it}) \tag{10}$$

is an index of all variable factors and ς_{it} is an iid error introduced by the fact that the first order conditions might not hold exactly. Applying all these results to Equation 1 we get

$$r_{it} = v i_{it} + \frac{\gamma}{\mu} k_{it} + \omega_{it} + \varsigma_{it} \tag{11}$$

where

$$\omega_{it} = \frac{1}{\mu}(a_{it} + \lambda_{it}) \tag{12}$$

There are three things to note about Equation 11: First, the error term is a composite of the idiosyncratic demand and the supply shock. They both represent genuine spread in the sense that – ceteris paribus – shifting resources from a plant with low λ_{it} or a_{it} to ones with higher values would make sense from a welfare point of view. I suggest therefore this composite measure of shocks as the relevant entity to seek in order to compute the true

spread[3]. I will refer to this as *Total Factor Value Productivity* (TFVP)
Second, efforts to estimate 11 will have to struggle with potential correlation between the unobserved shocks ω_{it} and the input variables vi_{it} and x_{Kit}. This is the classical production function endogeniety problem[4]. In plant level data correlation is introduced in addition through a between the exit decision of plants and the observed explanatory variables.
Third, an additional endogeneity problem is introduced by the measurement error problems discussed in the introduction; i.e. what we observe is not the true variable factor index vi_{it} but some distorted signal

$$\tilde{vi}_{it} = vi_{it} - \varrho_{it} \tag{13}$$

where ϱ_{it} is measurement error.
Point one suggested to get an estimate of ω_{it} in order to compute the true spread. Points two and three outlined the main obstacles in this effort. Next I will discuss how I address these obstacles. I will show that profits Π_{it} – defined as revenue less variable costs – can be used like an instrument for variable inputs. Equally, they can be used in a framework along the lines of Olley and Pakes [12] to account for the endogeniety problem.

[3] To make the point clearer consider the example of two plants producing fizzy drinks. Presidents Cola is US owned whereas Queens Cola is the British competitor. Suppose the two plants employ exactly the same production technology so that the number of cola tins produced per input i.e. a_{it} is equal across the two plants. Queens Cola can charge a higher price because their branding resonated with the patriotic feelings of Britons. If we only measured genuine TFP we would not capture this. Crucially the differential in the λ_{it} component suggests that - ceteris paribus - shifting production factors from Presidents to Queens Cola or allowing President Cola to sell using Queens Cola branding, will be welfare improving.

[4] Compare Grilliches and Mairesse[4]

2.1 How to account for endogeneity

Formally the endogeneity problem follows from the profit maximization problem of plants. If plants maximize profits conditional on the state variables capital k_{it} and plant specific demand and TFP shock composite, ω_{it}, then the variable factors are functions of k_{it} and ω_{it}: $l_{it} = l(k_{it}, \omega_{it})$ and $m_{it} = m(k_{it}, \omega_{it})$. Equally the firms short term profit function – i.e. revenue minus variable costs – is a function of ω_{it} and capital[5]:

$$\Pi_{it} = R_{it} - C_{it} = \Pi(\omega_{it}, k_{it}) \tag{14}$$

In the Appendix I show that under the assumptions made so far about market structure and production technology, this function is monotone in ω_{it}. This implies that we can invert it and write

$$\omega_{it} = \phi_{\omega}(\Pi_{it}, k_{it}) \tag{15}$$

where $\phi_{\omega}(\cdot) = \Pi^{-1}(\cdot)$. This means that we can use profits in a similar way as Olley and Pakes[12] have used investment and Levinsohn and Petrin[9] materials to control for ω_{it} in order to estimate 11. We start by assuming that ω_{it} evolves as a Markov process:

$$\omega_{it} = E\{\omega_{it} | \omega_{it-1}\} + \nu_{it} \tag{16}$$

where ν_{it} is iid. Consequently our regression equation 11 can be rewritten as

$$r_{it} - vi_{it} = \frac{\gamma}{\mu} k_{it} + E\{\omega_{it} | \omega_{it-1}\} + \nu_{it} + \varsigma_{it} \tag{17}$$

If we can assume that k_{it} is only correlated with the expected component of ω_{it} but not with ν_{it} then it is sufficient to control for $E\{\omega_{it}|\omega_{it-1}\}$ in order to estimate $\frac{\gamma}{\mu}$ consistently. Olley and Pakes get this condition by assuming that investment in t only affects the capital stock in $t+1$. An

[5]Formally all these variables are functions of factor costs and other aggregate variables as well, but to avoid notational clutter I focus on what varies between plants

alternative assumptions – which Olley and Pakes cannot make because they use investment in t to predict ω_{it} - is that investment in t s predetermined in period t.

But how to control for $E\{\omega_{it}|\omega_{it-1}\}$ after all? We do not know which function form $E\{\omega_{it}|\cdot\}$ takes but we have found in 15 a way to express its argument as a function of observables. We can therefore rewrite 11 as

$$r_{it} - vi_{it} = \frac{\gamma}{\mu}k_{it} + g(k_{it-1}, \Pi_{it-1}) + \nu_{it} + \varsigma_{it} \tag{18}$$

where $g(\cdot) = E\{\omega_{it}|\phi\omega(\cdot)\}$. If are willing to approximate $g(\cdot)$ by a higher order polynomial 18 reduces to a simple least squares problem. Alternatively we could use 18 to get initial values for a more challenging but more efficient – in the econometric sense – procedure: Start with a first stage nonparametric regression

$$r_{it} - vi_{it} = \phi(k_{it}, \Pi_{it}) + \varsigma_{it} \tag{19}$$

where $\phi(k_{it}, \Pi_{it}) = \frac{\gamma}{\mu}k_{it} + \phi_\omega(\Pi_{it}, k_{it})$ because as long as we do not know the functional form of $\phi_\omega(\cdot)$ we cannot identify $\frac{\gamma}{\mu}$ separately in such a regression. This provides an estimate $\hat{\phi}_{it}$ for each observation. Equation 18 can then be restated as a nonlinear least squares problem:

$$r_{it} - vi_{it} = \frac{\gamma}{\mu}k_{it} + h(\hat{\phi}_{it} - \frac{\gamma}{\mu}k_{it-1}) + \nu_{it} + \varsigma_{it} \tag{20}$$

where $h(\cdot) = E\{\omega_{it}|\cdot\}$ is approximated again by a third order polynomial.

2.2 Accounting for exit

The fact that input factors are functions of ω_{it} is not the only factor that leads to endogeneity in regressions of equation 11. Because we are working with plant level data and unlike e.g. countries, plants can exit the industry or die all together there is an additional endogeneity problem from a dependance of this exit decision on the current level of the capital stock. Ericson and Pakes[?] provide an elaborate model that formalises this idea. What is required intuitively is that the scrap value upon exiting increases slower than profits upon continuation with increasing capital stock. For the empirical application it suffices to note that there is some lower threshold level of ω which is a function of k_{it}

$$\underline{\omega}_{it} = \underline{\omega}_{it}(k_{it}) \tag{21}$$

If a plant i's level of ω_{it} drops below $\underline{\omega}_{it}$ it exits. Consequently our regression equation 22 becomes

$$r_{it} - vi_{it} = \frac{\gamma}{\mu}k_{it} + E\{\omega_{it}|\omega_{it-1}, \underline{\omega}_{it}\} + \nu_{it} + \varsigma_{it} \tag{22}$$

Thus to run this equation we need some form to control for $\underline{\omega}_{it}$ as well as for ω_{it}. I follow Olley and Pakes [12] and apply one of their derivations to my framework. Note that we can write for the probability that a plant exits:

$$\begin{aligned} P(\text{Stay after period t}) &= P(\omega_{it} > \underline{\omega}_{it+1}(k_{it+1})|\underline{\omega}_{it}(k_{it}), \omega_{it}) \\ &= p(\underline{\omega}_{it}(k_{it}), \omega_{it}) \\ &= p(k_{it}, \Pi_{it}) = P_{it} \end{aligned} \tag{23}$$

where the third equality follows from equation 15. Thus we can a run a Probit on exit with capital and profits as explanatory variables. This gives an estimate of P_{it}. Now if P_{it}, the probability that a plant stays in the market, increases monotonically with $\underline{\omega}_{it}$[6], $p(\cdot)$ is invertible so that we can write

$$\underline{\omega}_{it} = p^{-1}(P_{it}, k_{it}, \Pi_{it}) \tag{24}$$

which means that we can control for $\underline{\omega}_{it}$ using the estimate of $\P_{it}$. Consequently equation 18 becomes

$$r_{it} - vi_{it} = \frac{\gamma}{\mu}k_{it} + g(\Pi_{it-1}, k_{it-1}, \hat{P}_{it-1}) + \tilde{\nu}_{it} + \varepsilon_{it} \tag{25}$$

[6]not a very strong assumptions

and we can proceed as outlined in the last section.

2.3 Alternative approaches and difference to Olley and Pakes and Levinsohn and Petrin

see Martin[11]

2.4 The measurement error problem

Our measure of labour input is a simple headcount measure of the number of persons employed. This might hide important differences in the skill composition of the labour force between plants. Equally there might be variations in the quality of intermediate inputs between plants. Why is this a problem for TFP calculation? To gain some intuition I introduce a simple example: Suppose what matters for the plant is a composite measure of efficiency units of skilled and unskilled workers:

$$L_{it} = U_{it} + \phi H_{it} \tag{26}$$

where U_{it} denotes the number of unskilled and H_{it} the number of (highly) skilled workers and ϕ is a constant larger than one which denotes the relative productivity of high to low skilled workers. What we observe is simply the sum of skilled and unskilled workers of course:

$$\tilde{L}_{it} = U_{it} + H_{it}$$

The measurement error term becomes in this case

$$\varrho_{it} = s_L \varrho_{it}^L \tag{27}$$

where

$$\varrho_{it}^L = l_{it} - \tilde{l}_{it} = \frac{\phi H_{it}}{L_{it}}(h_{it} - \tilde{l}_{it}) + \frac{U_{it}}{L_{it}}(u_{it} - \tilde{l}_{it}) \tag{28}$$

If the two skill types enter the production function as suggested in equation 26 and relative wages reflect relative productivity, then the profit maximisation problem of plants does not determine the skill mix at a specific plant. Consequently plants would choose arbitrarily how many low or high skilled workers they employ. Now imagine that we have two plants with exactly the same level of ω_{it} and k_{it} but different choices regarding their skill mix. More specifically plant 1 decides to employ low skilled labour whereas plant 2 employs only high skilled labour. Because we assumed that their productivity is the same we must have that

$$l_{1t} = l(\omega_{1t}, k_{it}) = l_{2t} \tag{29}$$

i.e. their choice of effective labour input is the same. However, given their choices regarding the skill mix we have:

$$\tilde{l}_{1t} = l(\omega_{1t}) > \tilde{l}_{1t} = \frac{l(\omega_{1t})}{\phi} \tag{30}$$

i.e. the measured labour input of plant 1 is higher because it only employs less productive unskilled workers. Thus if we simply relied on measured labour input without further modification we would incorrectly conclude that TFVP in plant 1 is lower. Is there anything we can do to correct for this? My strategy is as follows: As suggested in equation 29, a result of profit maximisation is that the effective labour input is a function of ω_{it}. We do not observe ω_{it} but in the previous section I described how we can use profits Π_{it} as a predictor of ω_{it}. Crucially, the argument made there is not affected by the measurement error problem because profits are calculated as turnover minus labour costs and purchases which all are measured without error. So profits are correlated with the variable of interest and therefore comply with one of the requirements necessary to function as an instrument. Equally we need that they are not correlated with the measurement error. Is this the case? In Equation 28 I derived the relevant expression for the current example. Note that ϱ_{it}^L consists exclusively of expressions that refer to the relative skill mix: $\frac{\phi_{it}}{L_{it}}$ is the share of wage costs spend on high skilled labour. $u_{it} - \tilde{l}_{it}$ is the log of the share of unskilled workers. As argued before, with wages reflecting relative productivites the plant level skill mix is not determined by the profit maximization problem. Consequently, the error term cannot be correlated with neither ω_{it} nor Π_{it}. What if wages do not reflect relative productivity? In this case the skill mix is determined by the profit maximisation problem. However – depending on whether the relative wage is higher or lower than relative productivity – all plants whould do the same: either source all required labour from the pool of high skilled or all from the pool of low skilled. Thus ϱ_{it}^L would be constant across plants and again it is not correlated with ω_{it} or Π_{it}.

To address the measurement error problem I consequently suggest an instrumental variables procedure: in a first stage regress observed input quantities on a polynomial in Π_{it} and k_{it}.

$$\tilde{l}_{it} = \phi_L(\Pi_{it}, k_{it}) + \varrho_{it}^L \tag{31}$$

This yields and estimate of effective inputs quantities:

$$\hat{l}_{it} = \hat{\phi}_L(\Pi_{it}, k_{it}) \tag{32}$$

which is used in turn to calculate the variable factor index.

$$\hat{vi}_{it} = \sum_{z \neq K} \bar{s}_j(\hat{x}_{zit} - k_{it}) \tag{33}$$

I then proceed as described in Section 2.1 to get an estimate of $\frac{\gamma}{\mu}$ and $\phi(\cdot)$. Eventually I compute the corrected estimate of ω_{it} which underlies my calculation of the true spread as

$$\hat{\omega}_{it} = \hat{\phi}(\Pi_{it}, k_{it}) - \widehat{\left(\frac{\gamma}{\mu}\right)} k_{it} \tag{34}$$

An estimate of TFVP affected by measurement error I get as

$$\hat{\tilde{\omega}}_{it} = \tilde{vi}_{it} - \widehat{\left(\frac{\gamma}{\mu}\right)} k_{it} - \hat{\varsigma}_{it} \tag{35}$$

The formulation for effective labour input introduced in 26 is very restrictive. Why should low and high skilled labour be perfect substitutes? And maybe there are more than only two skill types. It turns out however that these assumptions are not needed to make the strategy just proposed work. In principle we can have infinitely many skill types. All that is needed is that the various input types enter as a homogenous composite index into the main production function. More specifically, suppose that the effective input of a variable production factor X_{it} is

$$X_{it} = \Xi(\mathbf{V}_{it}) \tag{36}$$

where $\Xi(\cdot)$ is a differentiable linear homogenous[7] function and $\mathbf{V}_{it}$ is a vector of N varieties of X_{it}. By linear homogeniety of $\Xi(\cdot)$ we can write

$$\ln \Xi(\mathbf{V}_{it}) = \sum_{z=1}^{N} \sigma_{zit} v_{zit} \tag{37}$$

where

$$\sigma_{zit} = \frac{\partial \ln \Xi(\mathbf{V}_{it})}{\partial \ln V_{zit}}$$

is equal to the share of variety z in payments to composite factor X_{it}. Now if instead of X_{it} we observe another linear homogenous composite of all varieties $\tilde{X}_{it} = \tilde{\Xi}\left(\mathbf{V}_{it}\right)$ – such as the total number of workers or the value of material inputs deflated by an industry price index – our measurement error becomes

$$\begin{aligned} x_{it} - \tilde{x}_{it} &= \textstyle\sum_{z=1}^{N} \sigma_{zit} \left(v_{zit} - \tilde{x}_{it}\right) \\ &= \textstyle\sum_{z=1}^{N} \sigma_{zit} \left(ln \left\{ \frac{V_{zit}}{\Sigma_{it}^{V}} \right\} - ln \left\{ \tilde{\Xi} \left(\frac{\mathbf{V}_{it}}{\Sigma_{it}^{V}} \right) \right\} \right) \end{aligned} \tag{38}$$

[7]The linear homogeniety assumption comes without loss of generality because we did not make any assumption regarding how X_{it} would enter the production function

where the second equality follows from linear homogeniety of $\tilde{\Xi}(\cdot)$ and

$$\Sigma_{it}^{V} = \sum_{z=1}^{N} V_{zit} \tag{39}$$

Consequently the error depends again only on relative indexes of the varieties which are either not determined by profit maximization or equal across plants by our assumption that factor prices are equalized across the economy. Therefore the error term is not correlated with profits and we can use a nonparametric regression of $\tilde{X}_{it}$ on profits and capital to get a consistent estimator of X_{it}.

2.5 Using wages to correct for measurement error

A common practice in production function regression to account for missing information on skills is to include the average wage level. This section examines in detail when this is a useful strategy and how it relates to my strategy put forward in the previous section. Assume again that the effective labour input is a function of N skill types as in equation 36. The average wage is thus equal to

$$W_{it} = \frac{\sum_{z=1}^{N} W_{zit} V_{zit}}{\tilde{L}_{it}} \tag{40}$$

where $\tilde{L}_{it} = \sum_{z=1}^{N} V_{zit}$. Taking logs and applying the mean value theorem yields

$$w_{it} = \sum_{z=1}^{N} \frac{W_{zit} V_{zit}}{\sum_{z=1}^{N} W_{zit} V_{zit}} (x_{zit} - \tilde{l}_{it}) \tag{41}$$

Note that if the relative wages for the various skill types correspond to the relative marginal productivities of all types actually employed[8] w_{it} is exactly equal to the expression for the measurement error in this case. How can we exploit this to get a measurement error free estimate of TFVP? Note first

[8]Some varieties might not be used because for any combination of inputs their marginal productivity is always lower than their wage.

that it is not sufficient to include the wage on the production function level as an additional regressor say in equation 18. As pointed out in equation 27 the error term at the level of the production function ϱ_{it} is the product of the actual measurement error in labour input and the share of the production factor in revenue. Consequently, if the revenue share does not happen to be constant as with a Cobb Douglas production function but instead varies with the amount of the input used, then inclusion of the wage at the level of equation 18 is not sufficient to control for the measurement error. Consequently – similar to the strategy described before – we have to correct the labour input we use, before computing the variable factor index vi_{it}. Since $w_{it} = \varrho_{it}^{L}$ the obvious way to do this is calculating an estimate of l_{it} as

$$\hat{l}_{it} = \tilde{l}_{it} + w_{it} \tag{42}$$

Note that this implies that we basically throw away the head count measure of labour as a variable because the resulting expression is equal to the log of total labour costs

$$\tilde{l}_{it} + w_{it} = \sum_{z=1}^{N} \frac{W_{zit}V_{zit}}{\sum_{z=1}^{N} W_{zit}V_{zit}} x_{zit} = TLC_{it}$$

where TLC_{it} represents total labour costs. Does this mean that both strategies – using ei ther a first stage regression of profits and capital or using average wages – are equally suitable to correct for the measurement error problem? Not necessarily. While the method using average wages seems somewhat simpler to compute because no additional regression step is needed it might be less efficient. Suppose there is an additional iid error affecting total labour costs so that we actually have

$$l_{it} = TLC_{it} + \varrho_{it}^{TLC} \tag{43}$$

One motivation for this could be that labour markets are not perfectly competitive so that wages at the firm level might idiosyncratically deviate from the rule that they represent relative marginal productivities. For consistency of both, the IV and the average wage procedure we equally need that this type of error is independent from everything else. However, whereas the average wage procedure would translate this error unfiltered into our estimate of the variable factor index, the IV procedure would smooth out some of it. In the results section I will show TFVP estimates using both procedures.

3 The Data

The source of my data is the Annual Respondents Database (ARD), the UK census of plants[9]. Not all UK plants are sent the ARD questionaire which includes the questions relevant for productivity analysis every year. Smaller plants are sampled on a random basis. Consequently, the set of usable observations consists of a reduced sample. The problem is eaggerbated by the fact that for the estimation procedure outlined in section 2 I require observations which are observed in consecutive years. The period covered by my sample are the years 1980 through 2000. Table 3 reports sample sizes along with descriptive statistics for selected years. Because the ONS increased the plant size threshold for random sampling of plants the sample size is somewhat lower in later years[10]

[9] More extensive descriptions of the ARD can be found in Barnes and Martin [**?**], Griffith [3] and Oulton[14]

[10] What happened is that although the threshold was increased the actual sample size increased because more plants were sampled. But as a consequence of this the share of plants in the sample which is not observed consecutively in the sample has increased. This creates a problem when calculating the capital stock, which is done via a perpetual inventory method (for details see Martin [10]). For the perpetual inventory method we ideally need to observe a plant in all years it exists. Because that's often not the case I interpolate investment figure – unless the number of missing years is too high. Because this last exclusion criterion applied to a larger fraction of plants in later years the number of usable observations dropped in later years.

Table 1: Descriptive statistics

year	obs	mean employment	mean $\frac{va}{l}$
1980	11712	346.35	19.14
1985	11109	296.32	23.04
1990	11085	279.03	26.53
1995	10734	228.65	33.44
1999	8958	188.15	33.85
2000	8632	182.60	33.88

4 Results

Table 4 reports spread calculations – defined as the log difference between the plant at the 90iest and the 10th percentile – for various productivity measures along with estimates of the coefficient on capital in equation 11, $\frac{\gamma}{\mu}$. All figures are averages from the 3 digit level at which all estimates are computed to 2 digit levels for simplicity. The last row reports averages for the economy as a whole. Consider column 2 first. We find that for the economy as whole the 90-10 difference is 1.077 log points which corresponds to a ratio of almost 3 to 1. In the food and beverages sector (15) the spread amounts to a ratio of 4 to 1. These are the spectacular differences discussed in the introduction[11]. What happens if we calculate TFVP instead? Column 6 contains the relevant calculations and column 8 the ratio between value added spread and TFVP spread. In almost all sectors the spread reduces considerably. Averaging accross the whole economy we find a 1.9 ratio between 90iest and 10th percentile plant which is about 60 percent of the value added spread (column 8). Only in one 2 digit sector – printing and publishing – TFVP shows a wider spread than value added.

Table 4 shows non aggregated results for the most extreme – in terms of TFVP spread – 3 digit sectors. The sectors with the highest TFVP spread –

[11]Note that the 5 to 1 ratio quoted in the introduction referred to gross output over employment whereas the figures here show value added over employment. Moreover, observations with negative value added were dropped.

Table 2: Averages across 2digit sectors

(1) Sector	(2) $\frac{\gamma}{\mu}$	(3) $\mathbf{va-l}$	(4) TFP	(5) $\omega+\varrho$	(6) ω	(7) %Meas.Err	(8) %Tr.Spr.
15	0.865	1.404	0.381	0.676	0.625	0.041	0.415
17	0.870	0.912	0.337	0.599	0.502	0.110	0.538
18	0.834	1.007	0.441	0.793	0.677	0.115	0.672
19	0.868	0.890	0.327	0.519	0.418	0.114	0.490
20	0.860	0.985	0.408	0.547	0.468	0.079	0.481
21	0.845	1.027	0.297	0.606	0.573	0.035	0.554
22	0.882	1.329	0.635	1.391	1.340	0.045	1.021
24	0.780	1.377	0.430	0.994	0.971	0.017	0.694
25	0.764	0.982	0.368	0.908	0.843	0.069	0.877
26	0.839	1.114	0.453	0.778	0.661	0.109	0.593
27	0.887	0.999	0.319	0.552	0.467	0.089	0.476
28	0.900	0.945	0.441	0.644	0.502	0.150	0.541
29	0.849	0.937	0.373	0.734	0.620	0.124	0.665
30	0.975	1.264	0.496	0.544	0.409	0.107	0.324
31	0.871	0.991	0.407	0.686	0.581	0.105	0.594
32	0.884	1.227	0.487	0.671	0.591	0.066	0.490
33	0.872	1.100	0.458	0.734	0.639	0.088	0.577
34	0.866	0.949	0.333	0.702	0.629	0.081	0.646
35	0.855	0.979	0.461	0.873	0.753	0.123	0.780
36	0.803	1.119	0.450	0.809	0.715	0.083	0.658
Total	0.858	1.077	0.407	0.725	0.636	0.090	0.589

All results were calculated at the 3 digit level and then averaged up to 2 digit sectors. Column 2 reports the estimated coefficient on the capital stock which is equal to $\frac{\gamma}{\mu}$. Columns 4 to 6 report the log difference between the 90iest and 10th percentile for various productivity measures. Column 7 is (5-6)/3. Column 8 is 6/3.

Table 3: The 5 sectors where measurement error is highest and lowest

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sectors with highest measurement error share							
Sector	**obs.**	$\frac{\gamma}{\mu}$	**(va − l)**	$\omega + \varrho$	ω	**%Meas.Error**	**%True spread**
296	264	0.964	0.859	0.521	0.285	0.275	0.331
262	1324	0.926	1.013	0.702	0.441	0.258	0.435
285	973	0.892	0.953	0.686	0.484	0.212	0.508
287	2994	1.010	1.001	0.537	0.329	0.208	0.328
173	1005	0.884	0.862	0.591	0.418	0.200	0.486
Sectors with lowest measurement error share							
Sector	**obs.**	$\frac{\gamma}{\mu}$	**(va − l)**	$\omega + \varrho$	ω	**%Meas.Error**	**%True spread**
159	2399	0.625	1.815	1.777	1.794	-0.009	0.988
223	85	1.244	1.706	1.620	1.627	-0.004	0.954
156	452	0.916	1.757	0.362	0.368	-0.004	0.210
264	508	0.619	1.083	1.224	1.226	-0.002	1.132
211	1131	0.820	1.119	0.673	0.669	0.004	0.597

reaching ratios of more than 6 to 1 – include pharmaceuticals (244), beverages (159), and reproduction of records (223). Sectors with particularly low spread include preparation and spinning of textiles fibres (176), tanning of leather (191) and manufacture of batteries (314).

How important is the correction for measurement error? Column 5 of table reports spread calculations for TVFP including the measurement error ϱ_{it} [12]. The resulting spread calculations are in the same range as that for ω alone. The calculations not corrected for measurement error lead to higher spread in most sectors however. Column 7 relates the difference measurement error correction makes to the spread in labour productivity[13]. For the economy as a whole this produces a ratio of about 10 percent. Table 4, which reports 3 digit sector results, shows that there is considerable hetero-

[12] see equation 27

[13] loosely referred to as the share of measurement error in labour productivity spread. Note that it is not based on formal decomposition of labour productivity spread

Table 4: The 5 sectors where the true spread is highest and lowest

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Sectors with highest true spread				
Sector	**obs.**	$\frac{\gamma}{\mu}$	$(\mathbf{y}-\mathbf{l})$	$(\omega+\varrho)$	ω	**%Meas.Error**	**%True spread**
244	1231	0.622	1.534	1.879	1.851	0.018	1.207
159	2399	0.625	1.815	1.777	1.794	-0.009	0.988
223	85	1.244	1.706	1.620	1.627	-0.004	0.954
222	3862	0.678	1.090	1.349	1.282	0.061	1.177
264	508	0.619	1.083	1.224	1.226	-0.002	1.132
			Sectors with lowest true spread				
Sector	**obs.**	$\frac{\gamma}{\mu}$	$(\mathbf{y}-\mathbf{l})$	$(\omega+\varrho)$	ω	**%Meas.Error**	**%True spread**
176	114	1.030	0.740	0.290	0.199	0.123	0.270
191	224	0.921	0.859	0.293	0.219	0.087	0.255
314	193	0.960	0.895	0.387	0.264	0.137	0.295
152	438	0.929	1.229	0.357	0.273	0.068	0.222
296	264	0.964	0.859	0.521	0.285	0.275	0.331

geneity in this number. For sectors such as manufacture of weapons (296), manufacture of non-refractory ceramic goods and metal treatment (285) the measurement error share reaches values of more than 20 percent. In sectors such as manufacture of beverages (159), reproduction of recorded media (223), manufacture of grain mill products (156) or manufacture of bricks and tiles the measurement error has a tendency to reduce measured spread.

Thus controlling for substitution between labour and other production factors is important to examine the productivity spread and measurement error has a tendency to increase measured productivity spread although it does not a dramatic impact.

4.1 Alternative TFP measures

How specific are these results to the particular choice of productivity measure? In this section I compare the TFVP measure introduced in section 2 firstly to a relative TFP measure calculated using factor shares, referred to as TFP and calculated as

$$tfp_{it} = r_{it} - \bar{s}_L l_{it} - \bar{s}_M m_{it} - (1 - \bar{s}_L - \bar{s}_M) k_{it} \tag{44}$$

where as before lower case letters denote log deviations from the median plant in terms of gross output in each 3 digit industry year cell and the factor shares used are the average between the factor share at plant i and at the median plant. The second measure I compute is TFVP with the correction for measurement error relying on the average wage as outlined in section 2.5, ω_{wage}. Thirdly, I calculate ω_{LP}, which is TFVP as in section 2 except that I predict ω_{it} using material inputs instead of profits. This corresponds to the idea of Levinsohn and Petrin adapted to the current context of imperfect competition.

Table 5: Correlation between various TFP measures

	(1) $va - l$	(2) TFP	(3) $\omega + \varrho$	(4) ω	(5) ω_{wage}	(6) ω_{LP}
$va - l$	1.000	0.652	0.676	0.531	0.532	0.304
TFP	0.652	1.000	0.340	0.327	0.413	0.011
$\omega + \varrho$	0.676	0.340	1.000	0.872	0.818	0.700
ω	0.531	0.327	0.872	1.000	0.938	0.816
ω_{wage}	0.532	0.413	0.818	0.938	1.000	0.690
ω_{LP}	0.304	0.011	0.700	0.816	0.690	1.000

$va - l$ is log value added per employee relative to the 3 digit median plant in a given year. TFP is relative to the median plant and calculated using a factor share method. $\omega + \varrho$ is TFVP without correcting for measurement error in labour input. ω_{wage} is TFVP where measurement error is controlled for using the average wage. ω_{LP} uses material inputs to control for ω_{it} which corresponds to the method of Levinsohn and Petrin[9] adopted to the current context.

Table 4.1 shows the correlations between these various measures also reporting the respective correlations for labour productivity $(va - l)$ and TFVP including the measurement error induced by labour input mismeasurement

$(\omega + \varrho)$. The main message from this table is that differences in methodology matter. In particular the correlation between TFP and the TFVP class of productivity measures are very low. The correlation between ω and TFP is merely 0.33. That's less than the correlation between ω and labour productivity: 0.53. The correlation between the three TFVP measure are rather high: ω and ω_{wage} have a correlation of 0.94, ω and ω_{LP} are with 0.82 somewhat less correlated.

How do the spread calculations with these alternative measures look like? Column 4 of table 4 reports numbers for TFP which turn out to be considerably lower than the same figures TFVP reported in the previous section. The average log difference between the 90iest and the 10th percentile plant is now 0.4 log points which corresponds to a ratio of 1.5. Is there an explanation for this? Note that the capital coefficient value reported in column 1 generally takes on values which are below 1. Given the interpretation of the coefficient as the ratio between the scale parameter γ and the markup parameter μ, this is consistent with a certain amount of imperfect competition and either constant or increasing returns to scale. An implicit assumption in the calculation of TFP is that this capital coefficient is equal to 1 which means that there is no imperfect competition and there are constant returns to scale. This means that if capital stock and ω are positively correlated, some of the difference in ω is attributed to the capital stock and consequently ω is underestimated.

Spread calculations for ω_{wage} and ω_{LP} along with capital coefficient estimates for these two cases are reported in table 4.1. The spread calculations are very similar. It appears however that spread measures based on ω are generally larger then the spread measures based on ω_{wage} or ω_{LP}. This pattern is however not uniform across 2 digit sectors nor is the ranking of sectors consistent across the 3 measures. The capital coefficient $\frac{\mu}{\gamma}$ is generally smaller for ω for the other two measures, but again there is no uniformity nor ranking consistence accross sectors. An explanation for why the spreads calculated using ω are generally higher than the ones based on ω_{wage} could be as follows: If there is an additional error such as the one introduced in equation 43 then in the ω_{wage} calculation all of it would be attributed to the measurement error in labour ϱ_L. Compared to the ω case that's an overestimation of ϱ_L which in turn leads to an underestimation of TFVP. In the ω_{LP} case on the other hand we use material inputs, m_{it}, instead of profits Π_{it} to predict ω_{it}. If m_{it} is less good a predictor than Π_{it} then this might well lead to and underestimation of ω_{it}.

5 Good or bad spread?

Section 4 showed that the productivity spread is considerably lower for TFVP than for plain labour productivity suggesting that the economically relevant magnitude of the spread is not as bad as thought initially. But even in terms of TFVP for the economy as a whole the best plants are on average more than 80 percent more productive than the worst[14]. Is this something to worry about because it means that poorly performing plants are not forced to exit or improve or is it simply a consequence of a healthy process of selection. A relevant statistic to answer this question is the persistence of this productivity dispersion over time.

Table 5 shows a 3 year transition matrix for the distribution of value added over employment; i.e. the cells of table 5 contain estimates of the probability that a plant that is in the bottom quintile in year t, say, moves to the second quintile in $t + 3$ (row 1 column 2). In for labour productivity this probability is 15 percent and calculated from the number of such transitions we observe in the course of my 1980-2000 sample. The last column of table 5 contains estimates of the probability that a plant exits between t and $t + 3$.

[14]Table 4, Column 6

Table 6: Averages across 2digit sectors

(Alternative ways to control for endogeneity and measurement error)

(1) Sector	(2) $\frac{\gamma}{\mu}_{wage}$	(3) $\omega_{\mathbf{wage}}$	(4) $\frac{\gamma}{\mu}_{LP}$	(5) $\omega_{\mathbf{LP}}$
15	0.867	0.583	0.873	0.492
17	0.887	0.445	0.874	0.439
18	0.881	0.450	0.839	0.606
19	0.887	0.346	0.884	0.328
20	0.874	0.407	0.839	0.439
21	0.891	0.412	0.870	0.420
22	0.809	0.809	0.968	1.670
24	0.787	0.882	0.806	0.784
25	0.762	0.787	0.812	0.590
26	0.867	0.548	0.818	0.632
27	0.906	0.353	0.907	0.334
28	0.883	0.498	0.921	0.316
29	0.864	0.527	0.846	0.532
30	0.960	0.373	0.912	0.348
31	0.877	0.454	0.871	0.497
32	0.858	0.569	0.869	0.567
33	0.875	0.545	0.871	0.502
34	0.854	0.658	0.855	0.621
35	0.845	0.712	0.881	0.523
36	0.830	0.567	0.830	0.544
Total	0.863	0.544	0.867	0.539

All results were calculated at the 3 digit level and then averaged up to 2 digit sectors. Column 2 and 3 report the results for the capital coefficient and the estimate of the spread when total labour costs are used as labour input measure. Columns 4 and 5 report the same statistics when intermediate inputs are used to predict ω_{it}. This corresponds to the methodology of Levinsohn and Petrin [9] adapted for the current context of imperfect competition and increasing returns to scale as well as controlling for measurement error in labour input.

Table 7: Transition matrix for $\frac{VA}{L}$

	20	40	60	80	100	exit
20	0.34	0.17	0.08	0.04	0.02	0.34
40	0.17	0.24	0.17	0.10	0.04	0.28
60	0.08	0.18	0.22	0.18	0.07	0.27
80	0.04	0.10	0.19	0.26	0.16	0.25
100	0.02	0.04	0.08	0.18	0.40	0.28
entry	0.22	0.19	0.18	0.19	0.21	0.00

The cells report estimates of transition probabilities; e.g. the cell in column 2 of row 1 reports what fraction of plants that were in the bottom quintile in a given year managed to move to the second quintile three years later. The exit column report what fraction exited over the three year intervall. The entry row reports how entering plants are distributed accross productivity quintiles. The switching of the 3 digit industry by a plant was treated as an exit with consecutive entry.

Table 8: Transition matrix for TFP

	20	40	60	80	100	exit
20	0.33	0.18	0.11	0.06	0.03	0.30
40	0.18	0.22	0.18	0.11	0.04	0.27
60	0.10	0.18	0.21	0.18	0.08	0.25
80	0.06	0.12	0.18	0.23	0.14	0.27
100	0.03	0.06	0.10	0.18	0.30	0.33
entry	0.22	0.18	0.17	0.19	0.24	0.00

see notes of table 5

Table 9: Transition matrix for TFVP

	20	40	60	80	100	exit
20	0.29	0.17	0.07	0.03	0.01	0.44
40	0.14	0.27	0.19	0.07	0.02	0.32
60	0.05	0.17	0.28	0.19	0.04	0.27
80	0.02	0.06	0.18	0.35	0.16	0.23
100	0.01	0.02	0.04	0.16	0.56	0.21
entry	0.29	0.20	0.18	0.17	0.16	0.00

see notes of table 5

What can table 5 tell us about the quality of the productivity spread? The striking about plant level productivity distribution is that it is persistent[15]. The diagonal elements of the matrix in table 5 are much higher than the off diagonal elements suggesting that plants are most likely to remain at their current position in the rather than move up or down. This result is core in suggestion that there might be a problem with the productivity spread. If there are plants in the market with much higher productivity, why are lagging plants not more likely to move up – or exit for that matter? Looking at the last column it is somewhat reassuring that the bottom 34 percent the bottom quintile plants have a higher exit probability than other plants. But then again even for the top plants the exit probability is with 28 percent fairly similar to the bottom one.

Before making any fast conclusions on all of this the key question to answer is if labour productivity captures plant performance and in turn dynamic behaviour adequately. Table 5 shows the transition matrix for TFP[16]. It turns out that persistence does not change much and exit probability of top plants is now actually higher than for bottom plants, which is rather implausible[17]. So what happens if we use TFVP instead? Table **??** , which has the relevant numbers, paints a much more positive picture of the productivity spread. Persistence of the bottom plants is 5 percentage points

[15]This is a result stressed by other authors before. Compare Baily et al.[1], Bartelsman and Dhrymes[2] or Haskel[6]

[16]as defined in equation 44

[17]In Haskel and Martin [5] TFP transitions matrices did not have this feature although exit probabilities for top and bottom were very close. Here I compute TFP slightly different however and plants with negative profits are dropped. In that paper we calculated the median in equation 44 for every production factor separately. This is not an entirely correct application of the mean value theorem on which the notion of relative TFP is based. What I do in this paper instead is identifying the median plant in terms of gross output and then use the input factor levels of this one plant to derive the relative expressions of the other variables.

lower and persistence of top plants 16 points higher compared to the labour productivity case. Exit probability of bottom plants is twice as high as that of top plants.

6 Conclusion

I put forward a novel way of computing TFP in this paper which combines a refined version of the methodology of Olley and Pakes[12] with the revenue production framework introduced by Klette and Griliches[7]. The framework allows for a very flexible production technology, non constant returns to scale, addresses the endogeneity of inputs problem in production function estimation and controls for measurement error in labour inputs.

Measurement error in labour inputs is found to increase measured productivity spreads although not dramatically. The TFP measure obtained with my method leads to productivity spread estimates which are on average about 60 percent the size of the labour productivity spread. Compared to labour productivity and even more so compared to TFP calculated in a standard way, the dynamic features of the resulting productivity measure are more plausible and more in line with the notion that productivity spreads are the static trace of a dynamic learning and selection process.

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A The monotone relationship between profits and shocks

Start by noting that given our assumption of a homogenous production function 2 we can write the cost minimization problem as

$$\tilde{C}(\tilde{K}_{it}, \mathbf{w}_{Vit}) = \min_{\tilde{\mathbf{X}}_{Vit}} \sum_{z \neq K} w_{zit} \tilde{X}_{zit} \text{ s.t. } 1 = f\left(\tilde{K}_{it}, \tilde{\mathbf{X}}_{Vit}\right) \tag{45}$$

where $\tilde{K}_{it} = \frac{K_{it}}{\tilde{Y}_{it}}$ with $\tilde{Y}_{it} = \left(\frac{Y_{it}}{A_{it}}\right)^{\frac{1}{\gamma}}$. $\tilde{\mathbf{X}}_{Vit}$ collects the same transformation for all variable production factors in a vector. Total cost become in terms of Equation 45

$$C_{it} = \tilde{C}_{it} \tilde{Y}_{it} \tag{46}$$

Next consider the profit function.

$$\Pi_{it}(K_{it}, \lambda_{it}, a_{it}, \mathbf{w}_{it}) = R_{it} - C_{it}$$

Given the demand function 5 and the cost function 46 we can write it as

$$\Pi_{it}(K_{it}, \lambda_{it}, a_{it}, \mathbf{w}_{it}) = \left(\frac{\Lambda_{it} R_t}{P_t}\right)^{\frac{1}{\eta}} P_t Q^{1-\frac{1}{\eta}} - \tilde{C}_{it} \tilde{Y}_{it} \tag{47}$$

Note that the firm's profit maximization first order condition is

$$\left(1 - \frac{1}{\eta}\right) \frac{R_{it}}{Q_{it}} = \frac{1}{\gamma} z(\tilde{Y}_{it}, \tilde{K}_{it}) \frac{\tilde{Y}_{it}}{Q_{it}} \tag{48}$$

where

$$z(\tilde{Y}_{it}, \tilde{K}_{it}) = \frac{\partial \tilde{C}_{it}}{\partial \tilde{Y}_{it}} \tilde{Y}_{it} + \tilde{C}_{it} \tag{49}$$

Finally, note that the derivatives of profit with respect to changes in λ_{it} and a_{it} are

$$\frac{\partial \Pi_{it}}{\partial \lambda_{it}} = \mu^{-1} R_{it}$$

and

$$\frac{\partial \Pi_{it}}{\partial a_{it}} = z(\tilde{Y}_{it}, \tilde{K}_{it}) \frac{1}{\gamma} \left(\frac{Q_{it}}{A_i t} \right)^{\frac{1}{\gamma}} = \mu^{-1} R_{it} \tag{50}$$

where the last equality follows from the first order condition 48[18] and

$$\mu = \left(1 - \frac{1}{\eta} \right)^{-1}$$

As a consequence of all these results we get for the total differential of profits

$$d\Pi_{it} = R_{it} \frac{1}{\mu} \left(d\lambda_{it} + da_{it} \right) = R_{it} d\omega_{it} \tag{51}$$

which establishes that there is a positive relationship between profits and composite shock index ω_{it}.

B Definitions

Table 10: 3 digit industry classification

Sector	**Description**
151	Production, processing and preserving of meat and meat products
152	Processing and preserving of fish and fish products
153	Processing and preserving of fruit and vegetables
154	Manufacture of vegetable and animal oils and fats
155	Manufacture of dairy products
156	Manufacture of grain mill products, starches and starch products
157	Manufacture of prepared animal feeds
158	Manufacture of other food products
159	Manufacture of beverages
160	Manufacture of tobacco products
171	Preparation and spinning of textile fibres

[18]This is an application of the envelope theorem

Table 10: (continued)

Sector	**Description**
172	Textile weaving
173	Finishing of textiles
174	Manufacture of made-up textile articles, except apparel
175	Manufacture of other textiles
176	Manufacture of knitted and crocheted fabrics
177	Manufacture of knitted and crocheted articles
181	Manufacture of leather clothes
182	Manufacture of other wearing apparel and accessories
183	Dressing and dyeing of fur; manufacture of articles of fur
191	Tanning and dressing of leather
192	Manufacture of luggage, handbags and the like, saddlery and harness
193	Manufacture of footwear
201	Saw milling and planing of wood, impregnation of wood
202	Manufacture of veneer sheets; manufacture of plywood, laminboard, particle board
203	Manufacture of builders' carpentry and joinery
204	Manufacture of wooden containers
205	Manufacture of other products of wood; manufacture of articles of cork, straw an
211	Manufacture of pulp, paper and paperboard
212	Manufacture of articles of paper and paperboard
221	Publishing
222	Printing and service activities related to printing
223	Reproduction of recorded media
231	Manufacture of coke oven products
232	Manufacture of refined petroleum products
233	Processing of nuclear fuel
241	Manufacture of basic chemicals
242	Manufacture of pesticides and other agro-chemical products
243	Manufacture of paints, varnishes and similar coatings, printing ink and mastics
244	Manufacture of pharmaceuticals, medicinal chemicals and botanical products
245	Manufacture of soap and detergents, cleaning and polishing preparations, perfume
246	Manufacture of other chemical products
247	Manufacture of man-made fibres
251	Manufacture of rubber products
252	Manufacture of plastic products
261	Manufacture of glass and glass products
262	Manufacture of non-refractory ceramic goods other than for construction purposes
263	Manufacture of ceramic tiles and flags
264	Manufacture of bricks, tiles and construction products, in baked clay
265	Manufacture of cement, lime and plaster
266	Manufacture of articles of concrete, plaster and cement
267	Cutting, shaping and finishing of stone
268	Manufacture of other non-metallic mineral products
271	Manufacture of basic iron and steel and of ferro-alloys (ECSC)
272	Manufacture of tubes
273	Other first processing of iron and steel and production of non-ECSC ferro-alloys
274	Manufacture of basic precious and other non-ferrous metals

Table 10: (continued)

Sector	Description
275	Casting of metals
281	Manufacture of structural metal products
282	Manufacture of tanks, reservoirs and containers of metal; manufacture of central
283	Manufacture of steam generators, except central heating hot water boilers
284	Forging, pressing, stamping and roll forming of metal; powder metallurgy
285	Treatment and coating of metals; general mechanical engineering
286	Manufacture of cutlery, tools and general hardware
287	Manufacture of other fabricated metal products
291	Manufacture of machinery for the production and use of mechanical power, except
292	Manufacture of other general purpose machinery
293	Manufacture of agricultural and forestry machinery
294	Manufacture of machine tools
295	Manufacture of other special purpose machinery
296	Manufacture of weapons and ammunition
297	Manufacture of domestic appliances not elsewhere classified
300	Manufacture of office machinery and computers
311	Manufacture of electric motors, generators and transformers
312	Manufacture of electricity distribution and control apparatus
313	Manufacture of insulated wire and cable
314	Manufacture of accumulators, primary cells and primary batteries
315	Manufacture of lighting equipment and electric lamps
316	Manufacture of electrical equipment not elsewhere classified
321	Manufacture of electronic valves and tubes and other electronic components
322	Manufacture of television and radio transmitters and apparatus for line telephon
323	Manufacture of television and radio receivers, sound or video recording or repro
331	Manufacture of medical and surgical equipment and orthopaedic appliances
332	Manufacture of instruments and appliances for measuring, checking, testing, navi
333	Manufacture of industrial process control equipment
334	Manufacture of optical instruments and photographic equipment
335	Manufacture of watches and clocks
341	Manufacture of motor vehicles
342	Manufacture of bodies (coachwork) for motor vehicles; manufacture of trailers an
343	Manufacture of parts and accessories for motor vehicles and their engines
351	Building and repairing of ships and boats
352	Manufacture of railway and tramway locomotives and rolling stock
353	Manufacture of aircraft and spacecraft
354	Manufacture of motorcycles and bicycles
355	Manufacture of other transport equipment not elsewhere classified
361	Manufacture of furniture
362	Manufacture of jewellery and related articles
363	Manufacture of musical instruments
364	Manufacture of sports goods
365	Manufacture of games and toys
366	Miscellaneous manufacturing not elsewhere classified
371	Recycling of metal waste and scrap
372	Recycling of non-metal waste and scrap

Table 10: (continued)

Sector	**Description**
401	Production and distribution of electricity
402	Manufacture of gas; distribution of gaseous fuels through mains
403	Steam and hot water supply
451	Site preparation
452	Building of complete constructions or parts thereof; civil engineering
453	Building installation
454	Building completion
455	Renting of construction or demolition equipment with operator